Two Field BPS Solutions for Generalized Lorentz Breaking Models

A. de Souza Dutra[a,b]*, M. Hott[b]† and F. A. Barone[b]‡
[a]Abdus Salam ICTP, Strada Costiera 11, Trieste, I-34100 Italy.
[b]UNESP-Campus de Guaratinguetá-DFQ§
Departamento de Física e Química
12516-410 Guaratinguetá SP Brasil

February 1, 2008

Abstract

In this work we present non-linear models in two-dimensional space-time of two interacting scalar fields in the Lorentz and CPT violating scenarios. We discuss the soliton solutions for these models as well as the question of stability for them. This is done by generalizing a model recently published by Barreto and collaborators and also by getting new solutions for the model introduced by them.

PACS numbers: 11.15.Kc, 11.27.+d

*e-mail: dutra@feg.unesp.br
†e-mail: hott@feg.unesp.br
‡e-mail: fbarone@feg.unesp.br
§Permanent Institution

Some years ago, Carrol et al [1] start to analyze the problem of Lorentz breaking signature in field theoretical models. By now, there are a great number of works discussing a kind of symmetry breaking in many different physically interesting contexts. For instance, in [2] it was discussed some impact over the standard model of this kind of symmetry breaking. Azatov *et al* [3], in a recent work, have analyzed the spontaneous breaking of the four dimensional Lorentz invariance of the QED through a nonlinear vector potential constraint, Bezerra *et al* [4] have shown that a space-time with torsion interacting with a Maxwell field by means of a Chern-Simons like term can explain the optical activity in the sinchroton radiation emitted by cosmological distant radio sources, Lehnert *et al* [5] have verified the consequences over the Cerenkov effect of a Lorentz-violating vacuum and Bluhm [6] has made an estimative analysis of the Lorentz and CPT bounds attainable in Penning-trap experiments. In fact, along the last years a considerable effort has been drawn into this direction by many groups and in a variety of physical applications. On the other hand, the presence of topological solutions of nonlinear models is a matter of large interest and possible applications [7, 8, 9]. On the other hand, a natural place to apply these ideas is that of condensed matter non-relativistic ground, where the break of isotropy and homogeneity emerges quite naturally, due the material structure.

As a consequence of the above arguments, it is natural to look for topological structures in CPT breaking scenarios. In fact, in a very recent work in this journal, Barreto *et al* [10] have introduced an approach capable of getting kinks in CPT violating scenarios.

Here we are going to discuss a generalization of the work of reference [10], both by obtaining more general solutions for the models considered on that work and by generalizing Lorentz breaking Lagrangian densities. Particularly we obtain solutions which were absent in the reference [10]. For this last accomplishment, we use a method recently introduced by one of us [11].

Models with Lorentz breaking terms usually leads to non-linear differential equations, and one of the problems appearing as a consequence of this nonlinearity is that, in general, we loose the capability of getting the complete solutions. Here we extend an approach exposed in reference [11] which shows that for some two field systems in 1+1 dimensions, whose the second-order differential equations can be reduced to the solution of corresponding first-order equations (the so called Bolgomol'nyi-Prasad-Sommerfield (BPS) topological solitons [12]), one can obtain a differential equation relating the two coupled fields which, once solved, leads to the general orbit connecting the vacua of the model. In fact, the "trial and error" methods historically arose as a consequence of the intrinsic difficulty of getting general methods of solution for nonlinear differential equations. About two decades ago, Rajaraman [7] introduced an approach of this nature for the treatment of coupled relativistic scalar field theories in 1+1 dimensions. His procedure was model independent and could be used for the search of solutions in arbitrary coupled scalar models in 1+1 dimensions. However, the method is limited in terms of the generality of the solutions obtained and is con-

venient and profitable only for some particular, but important, cases [13]. Some years later, Bazeia and collaborators [14] applied the approach developed by Rajaraman to special cases where the solution of the nonlinear second-order differential equations are equivalent to the solution of corresponding first-order nonlinear coupled differential equations. In this work we are going to present a procedure which is absolutely general when applied to Lorentz and CPT breaking systems, like those obtained from an extension of the ones described in [11] applied to nonbreaking versions appeared in [14]-[19]. Furthermore, we also show that many of these systems can be mapped into a first-order linear differential equation and, as a consequence, can be solved in order to get the general solution of the system. After that, we trace some comments about the consequences coming from these general solutions.

1 BPS nonlinear Lorentz and CPT scenarios

The two field model we shall study in $1+1$ dimensions is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \chi)^2 - f^\mu(\phi,\chi)\partial_\mu \chi - g^\nu(\phi,\chi)\partial_\nu \phi - V(\phi,\chi), \tag{1}$$

where $\mu = 0, 1$, $f^\mu(\phi,\chi)$ and $g^\nu(\phi,\chi)$ are vector functions with a prescribed functional dependence on the dynamical fields ϕ and χ, and $V(\phi,\chi)$ is a potential term.

Note that we can recover some usual Lorentz symmetry breaking models from (1) by choosing appropriately the vectors f^μ and g^μ. In particular, if $f_1 = s_2\,\phi$ and $g_1 = s_1\,\chi$, one recovers the model introduced very recently by Barreto and collaborators [10]. In fact, the first example we work out here is precisely this one, which we are going to show possesses an entire topological sector not considered in the work of reference [10].

If the potential $V(\phi,\chi)$ can be written in such a way that

$$V(\phi,\chi) = \frac{1}{2}\left(\frac{dW(\phi,\chi)}{d\phi} - g_1(\phi,\chi)\right)^2 + \frac{1}{2}\left(\frac{dW(\phi,\chi)}{d\chi} - f_1(\phi,\chi)\right)^2 , \tag{2}$$

with $W(\phi,\chi)$ being any function of ϕ and χ, the energy density of the BPS states becomes

$$\mathcal{E}_{BPS} = \frac{1}{2}\left(\frac{d\phi}{dx} - \frac{dW(\phi,\chi)}{d\phi} + g_1(\phi,\chi)\right)^2 + \frac{1}{2}\left(\frac{d\chi}{dx} - \frac{dW(\phi,\chi)}{d\chi} + f_1(\phi,\chi)\right)^2 + \frac{dW}{dx}, \tag{3}$$

with $dW/dx = W_\phi \phi' + W_\chi \chi'$, where we have defined $W_\phi \equiv \frac{\partial W}{\partial \phi}$, $W_\chi \equiv \frac{\partial W}{\partial \chi}$ and the prime stands for space derivative.

From equation (3), we can see that the solutions of minimal energy are obtained from the following two coupled first order equations

$$\begin{aligned} \phi' &= W_\phi(\phi,\chi) - g_1(\phi,\chi) , \\ \chi' &= W_\chi(\phi,\chi) - f_1(\phi,\chi) , \end{aligned} \tag{4}$$

Finally the BPS energy is written, as usual, by

$$E_{BPS}=\left|W\left(\phi_j,\chi_j\right)-W\left(\phi_i,\chi_i\right)\right|\,, \tag{5}$$

where ϕ_i and χ_i mean the $i-th$ vacuum states of the model. Here, it is important to remark that the BPS solutions settle into vacuum states asymptotically. In other words, the vacuum states act as implicit boundary conditions of the BPS equations.

It is interesting to notice that in the first order equations of motion (4) and in the energy density (3) only the space components of the functional vectors f_μ and g_μ, f_1 and g_1 respectively, are present.

From now on, in order to solve the equations (4), let us consider models for which we can write ϕ as a function of χ, that is, $\phi(\chi)$. In this situation, instead of applying the usual trial-orbit approach [14]-[19], we note that it is possible to write the following equation

$$\frac{d\phi}{W_\phi-g_1}=dx=\frac{d\chi}{W_\chi-f_1}, \tag{6}$$

where the differential element dx is a kind of invariant. In these cases one is lead to

$$\frac{d\phi}{d\chi}=\frac{W_\phi-g_1}{W_\chi-f_1}. \tag{7}$$

Equation (7) is the generalization of the one studied in [11] to the case of nonlinear Lorentz and CPT breaking scenarios. It is, in general, a nonlinear differential equation relating the scalar fields of the model. If one is able to solve it completely for a given model, the function $\phi\left(\chi\right)$ can be used to eliminate one of the fields, so rendering the equations (4) uncoupled and equivalent to a single one. Finally, this uncoupled first-order nonlinear equation can be solved in general, even if numerically.

We have found this method simpler than the method of the orbits broadly and successfully applied to study the mapping of the soliton solutions and defect structures in problems involving the interaction two scalar fields. Despite of being simpler, the method applied here furnishes not only the same orbits than those obtained by using the method of the orbits appearing in the references [14]-[19], but also some new ones as can be seen through a comparison with reference [11]. In the example worked out below one can verify that, this time, the mapping constructed here furnishes the very same orbits obtained in the reference [10]. Notwithstanding, we were able to find new solitonic configurations, not observed by Barreto and collaborators.

2 The example of linear Lorentz and CPT breaking

In this section we consider the particular model introduced in the work of Barreto *et al* [10] in order to apply the method discussed in the previous section. In fact, we show in this example that the equation (7) can be mapped into a linear differential equation,

from which it is possible to obtain the general solutions for the soliton fields. In the case on the screen, the superpotential [10] is written as

$$W\left(\phi,\chi\right)=\ \phi-\frac{1}{3}\phi^{3}-r\,\phi\,\chi^{2}, \tag{8}$$

and the Lorentz symmetry breaking terms in the lagrangian density (1) are chosen to be given by $f_1(\phi,\xi)=s_2\,\phi$ and $g_1(\phi,\xi)=s_1\,\chi$, such that equation (7) is rewritten as

$$\frac{d\phi}{d\chi}=\frac{(\phi^{2}-1)+r\;\chi^{2}+s_{1}\,\chi}{2\,r\,\phi\,\chi+s_{2}\;\phi}\ , \tag{9}$$

where s_1 and s_2 are constants.

At this point one can verify that, performing the transformations

$$\chi=\zeta-\frac{s_{2}}{2\,r}, \tag{10}$$

and

$$\phi^{2}=\rho+1+\frac{s_{2}}{4\,r}\left(2\,s_{1}-s_{2}\right), \tag{11}$$

the equation (9) becomes

$$\frac{d\rho}{d\zeta}-\frac{\rho}{r\,\zeta}\ =\ \zeta\ -\frac{b}{r}, \tag{12}$$

which is a typical inhomogeneous linear differential equation [11]. The general solutions for the orbit equation are then easily obtained, giving

$$\phi^{2}-1=c_{0}\,\zeta^{\frac{1}{r}}+\frac{r}{2\,r-1}\;\zeta^{2}-\frac{b}{r-1}\,\zeta+k\qquad\text{for } r\neq 1 \text{ and } r\neq\frac{1}{2}, \tag{13}$$

$$\phi^{2}-1=-b\;\zeta\;\ln\left(\zeta\right)+c_{1}\,\zeta+\zeta^{2}+k,\qquad\text{for } r=1 \tag{14}$$

and

$$\phi^{2}-1=\zeta^{2}\ln\left(\zeta\right)+b\;\zeta+c_{2}\,\zeta^{2}+k,\qquad\text{for } r=\frac{1}{2}, \tag{15}$$

where $k\ \equiv\ \frac{s_2}{4\,r}\left(2\,s_1-s_2\right)$, $b\ \equiv\ s_2-s_1$ and c_0, c_1 and c_2 are arbitrary integration constants.

In general it is not possible to solve χ in terms of ϕ from the above solutions, but the contrary is always granted. Here, with the aid of (8) and (10), we shall substitute the expressions of $\phi\left(\chi\right)$ obtained from (13), (14) and (15) in the second equation (4), obtaining respectively:

$$\frac{d\zeta}{dx}=\pm\,2\,r\,\zeta\sqrt{1+c_{0}\,\zeta^{\frac{1}{r}}+\frac{r}{2\,r-1}\;\zeta^{2}-\frac{b}{r-1}\,\zeta+k}\,,\quad\text{for } r\neq 1,\ r\neq\frac{1}{2}, \tag{16}$$

$$
\begin{aligned}
\frac{d\zeta}{dx} &= \pm 2\, r\, \zeta \sqrt{1 - b\,\zeta\, \ln(\zeta) + c_1\, \zeta + \zeta^2 + k}\,, \quad \text{for } r = 1, \\
\frac{d\zeta}{dx} &= \pm 2\, r\, \zeta \sqrt{1 + \zeta^2 \ln(\zeta) + b\, \zeta + c_2\, \zeta^2 + k}\ , \quad \text{for } r = \frac{1}{2}.
\end{aligned}
\tag{17}
$$

Barreto and collaborators [10] have limited themselves to the orbits in which $r \neq 1$ and $r \neq 1/2$ and the arbitrary constant c_0 equals to zero or infinity. In the particular case with $c_0 = 0$ they have found a lump-like profile for the field $\chi(x)$ and a kink-like profile for the field $\phi(x)$. By integrating the equation (16) and substituting its solutions into the equation (10) we get the following solutions for the field $\chi(x)$

$$
\chi^A_\pm(x) = \frac{4\sqrt[3]{A} e^{\mp 2\sqrt{A} r (x - x_0)}}{(\sqrt{A} e^{\mp 2\sqrt{A} r (x - x_0)} + C)^2 - 4AB} - \frac{b}{2r}, \tag{18}
$$

where x_0 is a constant of integration, $A = 1 - b^2/4r$, $B = r/(2r-1)$, $C = b/(r-1)$ and we have taken $s_1 = 0$. On its turn the solutions for the field $\phi(x)$ are obtained by substituting the classical solutions of the equation (16) into the equation (13), namely

$$
\phi^A_\pm(x) = \pm \frac{\sqrt{A}[A e^{\mp 4\sqrt{A} r (x - x_0)} - (C^2 - 4AB)]}{(\sqrt{A} e^{\mp 2\sqrt{A} r (x - x_0)} + C)^2 - 4AB}\ . \tag{19}
$$

The above solutions are valid if the parameters satisfy the conditions $A > 0$ and $C^2 \neq 4AB$. The behavior of the above solutions are plotted in the figure 1 for the parameters $r = 0.4$ and $b = 0.6$. One can observe that in both pairs of solutions, (ϕ_+, χ_+) and (ϕ_-, χ_-), the field $\chi(x)$ exhibits a lump-like profile and the field $\phi(x)$ a kink-like profile. This behavior is also found in many systems of two interacting solitons reported in the literature.

More recently [11] it has been shown that many models of two interacting solitons, very similar to this one with explicit Lorentz symmetry breaking that we are presenting here, can also exhibit kink-like behavior for both of the soliton fields, depending on the range of the parameters of the model. Inspired on this achievement, we have shown that it is also possible to have kink-like profiles for both of the fields, for particular values of the parameters r and b, in the model treated here. In fact if one takes $b = 2(r-1)/\sqrt{r}$, which corresponds to one of the solutions with $C^2 = 4AB$, and $r > 1/2$ in the equations (18) and (19) we obtain the following forms for the fields

$$
\chi^B_\pm(x) = \frac{4(2r-1)}{r\left(\sqrt{2r-1}\ e^{\mp 2\sqrt{2r-1}(x - x_0)} + 4\sqrt{r}\right)} - \frac{r-1}{r\sqrt{r}}, \tag{20}
$$

and

$$
\phi^B_\pm(x) = \pm \frac{4(2r-1)}{r\left(\sqrt{2r-1}\ e^{\pm 2\sqrt{2r-1}(x - x_0)} + \sqrt{2r-1}\right)}\ . \tag{21}
$$

In the figure 2 we present the behavior of the above kink solutions for $r = 2$.

One could interpret these solutions as representing two kinds of torsion in a chain, represented through an orthogonal set of coordinates ϕ and χ. So that, in the plane (ϕ,χ), the type-A kink corresponds to a complete torsion and the type-B corresponds to a half torsion, similarly to what has been done in [11].

It is worth mentioning that the pairs of type-B solutions have a BPS energy lower than that associated to the type-A soliton solutions. This can be shown by substituting the asymptotic values of the solutions in the equation (5), that is, for the type-A solutions we find $E_{BPS}^{A} = \frac{4}{3}A\sqrt{A}$, and $E_{BPS}^{B} = \frac{2}{3}A\sqrt{A}$ for the type-B solutions.

3 Generalized models

In what follows, we will study a more general model contemplating a number of particular cases which have been studied in the literature, including the previous and some other new ones. For this, we begin by defining the superpotential

$$W\left(\phi,\chi\right) = \frac{\mu}{2}\,\phi^{N}\,\chi^{2} + F\left(\phi\right)\ , \tag{22}$$

such that the equation (7) is given by

$$\frac{d\phi}{d\chi} = \frac{F_{\phi} + \frac{\mu}{2}\,N\ \phi^{(N-1)}\,\chi^{2} - g_{1}(\phi,\chi)}{\mu N\ \phi^{N}\,\chi - f_{1}(\phi,\chi)}, \tag{23}$$

where $F_{\phi} = dF/d\phi$. The space-component of the functionals terms responsible for breaking the Lorentz symmetry explicitly, namely, $f_1(\phi,\chi)$ and $g_1(\phi,\chi)$, are to be chosen more general than those of the model discussed previously and conveniently such that the integration of the equation (23) be possible. Based on the succesfull generalization of models of interacting solitons also carried out in the reference [11] and in the development of the model of the previous section, a possible generalized model can be constructed by choosing

$$F(\phi) = \frac{1}{2}\phi^{N}\left(\frac{\lambda}{N+2}\phi^{2} + \frac{\gamma}{N}\right)\ , \tag{24}$$

and the following forms for the functionals $f_1(\phi,\chi)$ and $g_1(\phi,\chi)$,

$$\begin{aligned} f_1(\phi,\chi) &= b\,\phi^{N}\chi\ , \\ g_1(\phi,\chi) &= a\,\phi^{N-1}\chi\ , \end{aligned} \tag{25}$$

where N is a positive integer number, λ and γ are constants and the parameters a and b can be thought as space-components of two-vectors pointing out in some preferred direction in space-time and the responsible for breaking the Lorentz symmetry.

The corresponding equation for the dependence of the field ϕ as a function of the field χ is now given by

$$\frac{d\phi}{d\chi}=\frac{1}{2}\frac{\mu\,N\;\phi^{N-1}\,\chi^2+\phi^{N-1}(\lambda\phi^2+\gamma)-2a\phi^{N-1}\chi}{\mu\,\phi^N\,\chi-b\phi^N}\;. \tag{26}$$

Now, by performing the transformations

$$\sigma=\frac{1}{2\mu}\left(\lambda\phi^2+\frac{Nb^2}{\mu}+\gamma-\frac{2ab}{\mu}\right)\;, \tag{27}$$

and

$$\varsigma=\mu\chi-b, \tag{28}$$

we get

$$\frac{d\sigma}{d\varsigma}-\frac{\lambda\sigma}{\mu\varsigma}=\frac{N\lambda}{2\mu^3}\varsigma+\frac{\lambda}{\mu^3}(Nb-a)\;. \tag{29}$$

The above equation is very similar to the equation (12) and can be easily integrated out. Its general solution in the case $\lambda\neq\mu$ and $\lambda\neq 2\mu$ is

$$\sigma\,(\varsigma)=\frac{\lambda(Nb-a)}{\mu^2(\mu-\lambda)}\varsigma+\frac{N\,\lambda}{2\,\mu^2\,(2\mu-\lambda)}\varsigma^2+c\;\varsigma^{\frac{\lambda}{\mu}}, \tag{30}$$

where c is an arbitrary integration constant. The solutions for the equation (29) in the cases $\lambda=\mu$ and $\lambda=2\mu$ can also be obtained, but we will not deal with them here.

We substitute the equations (27), (28) and (30) in one of the equations (4) to obtain the following first-order equation of motion for the field ς

$$\frac{d\varsigma}{dx}=\pm\mu^{1-N/2}\varsigma\left[\frac{N}{2\mu-\lambda}\varsigma^2+\frac{2(Nb-a)}{\mu-b}\varsigma+\frac{c\mu}{\lambda}\varsigma^{\lambda/\mu}-Nb^2-\gamma\mu+2ab\right]^{N/2}. \tag{31}$$

This last equation can be solved analytically or numerically, depending on the values of the parameters. For the particular case with $N=2$, $2b=a$ and $c=0$ we obtain very simple kink solutions for both of the fields $\phi(x)$ and $\chi(x)$, as can be verified from the behavior of the solution for the field $\varsigma(x)$

$$\varsigma(x)=\pm\frac{\sqrt{B}e^{Bx}}{\sqrt{1+Ae^{2Bx}}}\;, \tag{32}$$

where $A=2/(2\mu-\lambda)>0$ and $B=(\gamma\mu-2b^2)/\lambda>0$, and by substituting (32) in the equations (28), (30) and (27).

The construction of an even more general model which includes non-linear dependence on the field $\chi(x)$ can be carried out by following the generalization proposed

in the reference [11]. This can be accomplished by choosing the following form of the superpotential

$$W_{NM}\left(\phi,\chi\right)=\frac{\mu}{M}\,\phi^{N}\,\chi^{M}+F\left(\phi\right), \tag{33}$$

where $F(\phi)$ is given by the equation (24) and M is a positive integer. In order to include the terms responsible for breaking the Lorentz symmetry and to obtain a solution for the differential equation (7) it is reasonable to choose the functionals $f_1(\phi,\chi)$ and $g_1(\phi,\chi)$ in the following forms

$$\begin{aligned} f_1(\phi,\chi) &= b\,\phi^{N}\chi^{M-1} \\ g_1(\phi,\chi) &= a\,\phi^{N-1}\chi^{M}\ . \end{aligned} \tag{34}$$

With this generalization the equation (7) can be written in the form

$$\frac{d\varphi}{d\chi}-\frac{\varphi}{(\mu-b)\lambda}\chi^{1-M}=\frac{(\mu N-2aM)}{M(\mu-b)\lambda}\chi\ , \tag{35}$$

where $\varphi=\lambda\phi^2+\gamma$.

The equation (35) is similar to the one which appears in reference [11]. It admits the solution

$$\begin{aligned} \varphi(\chi) &= \exp\left[-\frac{1}{\lambda(\mu-b)}\frac{1}{M-2}\chi^{(2-M)}\right]\times \\ &\quad \left[\tilde{c}_1+\frac{2^{M/(M-2)}}{M(M-2)}\frac{\mu N-2aM}{2\lambda(\mu-b)}\chi^2\left(\frac{\chi^{(2-M)}}{(M-2)}\right)^{2/(M-2)}\times\right. \\ &\quad \left.\Gamma\left(\frac{2}{(M-2)},\frac{1}{\lambda(\mu-b)}\frac{\chi^{(2-M)}}{(M-2)}\right)\right] \end{aligned} \tag{36}$$

where $\tilde{c}_1$ is an arbitrary integration constant and $\Gamma(a,z)=\int_z^\infty t^{a-1}e^{-t}dt$ is the incomplete Gamma function.

4 Conclusions

We have been able to generalize a model presented recently in the reference [10] which incorporates the phenomena of solitons interactions and the Lorentz symmetry breaking. The generalization has been carried out in two ways. We have found non-trivial classical solutions which exhibit kink-like behavior for both of the interacting fields and, consequently, with BPS energy lower than that associated with the usual solutions presented previously for the same model. Another interesting aspect of the

kink-like solutions rest on the study of the stability of the solutions against small time-dependent linear perturbation. At least for some models with only one scalar field, it has been shown in the reference [20] that models with kink-like solutions possess the stability of these solutions, on the other hand, models with lump-like classical solutions are unstable. For two interacting scalar fields the problem is cumbersome, even though the authors of the reference [10] have been able to show, based on very elegant and general arguments, that the solutions found there, even with lump-like configurations for one of the fields, are stable. We understand that the analysis of the stability carried out in reference [10] is valid for reference systems in which $b_0 = 0$, where b_0 is the time-component of the two-vector responsible for the Lorentz symmetry breaking. For reference systems in which $b_0 \neq 0$ the analysis has not been done.

We have also proposed generalizations of the model of the reference [10] by introducing non-linear terms that break the Lorentz symmetry. This last generalization was possible thanks to the successful generalization carried out in the reference [11] which deals with a Lorentz symmetric two-dimensional model of interacting scalar fields.

Acknowledgements: The authors ASD and MH thanks to CNPq and FAB to FAPESP for the financial support. We also thanks to Professor D. Bazeia for introducting us to this matter and to the reference [10]. This work has been finished during a visit of ASD within the Associate Scheme of the Abdus Salam ICTP.

References

[1] S. Carrol, G. Field and R. Jackiw, Phys. Rev. D **41** (1990) 1231.

[2] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55** (1997) 6760; **58** (1998) 116002.

[3] A. I. Azatov and J. L. Chkareuli, Phys. Rev. D **73** (2006) 065026.

[4] V. B. Bezerra, C. N. Ferreira and J.A. Helayel-Neto, Phys. Rev. D **71** (2005) 044018.

[5] R. Lehnert and R. Potting, Phys. Rev. D **70** (2004) 125010.

[6] R. Bluhm, Phys. Rev. D **57** (1998) 3932.

[7] R. Rajaraman, *Solitons and Instantons* (North-Holand, Amsterdam, 1982)

[8] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge, Cambridge, UK, 1994).

[9] D. Walgraef, *Spatio-Temporal Pattern Formation* (Springer-Verlag, New York, 1997).

[10] M. N. Barreto, D. Bazeia and R. Menezes, Phys. Rev. D **73** (2006) 065015.

[11] A. de Souza Dutra, Phys. Lett. B **626** (2005) 249.

[12] M. K. Prasad and C. M. Sommerfield, Phys. Rev. Lett. **35** (1975) 760; E. B. Bolgomol'nyi, Sov. J. Nucl. Phys. **24** (1976) 449.

[13] L. J. Boya and J. Casahorran, Phys. Rev. A **39** (1989) 4298.

[14] D. Bazeia, M. J. dos Santos and R. F. Ribeiro, Phys. Lett. A **208** (1995) 84.

[15] D. Bazeia and F. A. Brito, Phys. Rev. Lett. **84** (2000) 1094.

[16] D. Bazeia and F. A. Brito, Phys. Rev. D **61** (2000) 105019.

[17] D. Bazeia, R. F. Ribeiro and M. M. Santos, Phys. Rev. E **54** (1996) 2943.

[18] D. Bazeia and E. Ventura, Chem. Phys. Lett. **303** (1999) 341.

[19] D. Bazeia, V. B. P. Leite, B. H. B. Lima and F. Moraes, Chem. Phys. Lett. **340** (2001) 205.

[20] G. Flores-Hidalgo and N. F. Svaiter, Phys. Rev D **66** (2002) 025031.

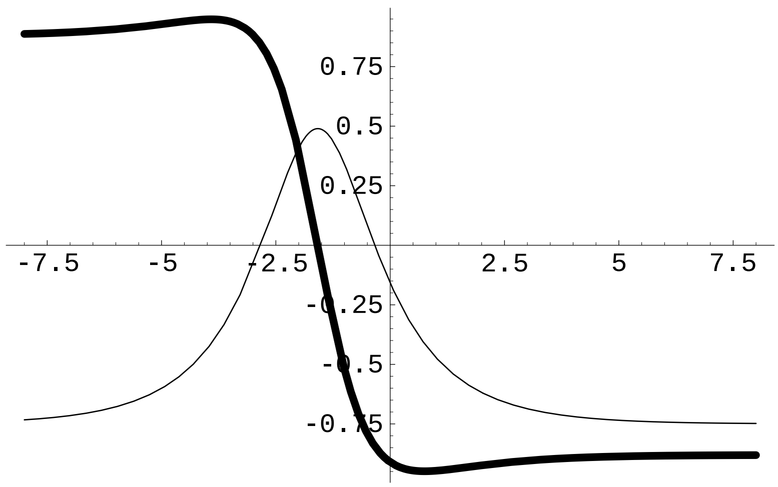

Figure 1: Typical type-A kink profile (for $r = 0.6$, $b = 0.4$). The thin line corresponds to the field $\chi_+(x)$ and the thick line to the field $\phi_+(x)$. Both were calculated for $c_0 = 0$.

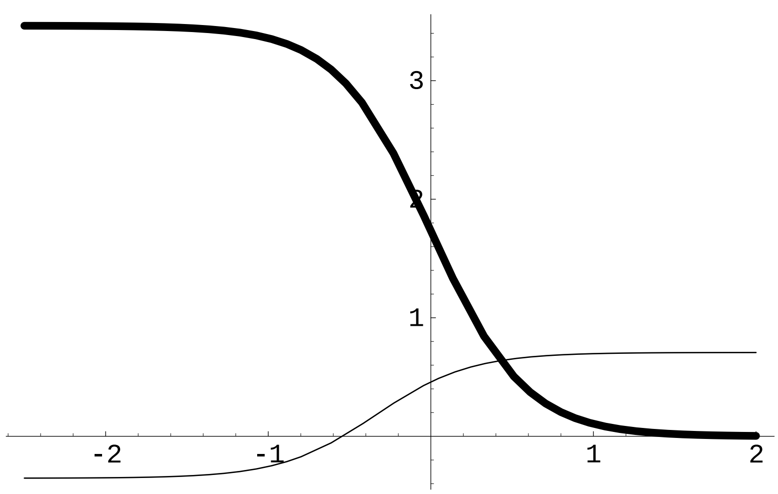

Figure 2: Typical type-B kink profile (for $r = 2$). The thin line corresponds to the field $\chi_+(x)$ and the thick line to the field $\phi_+(x)$. Both were calculated for $c_0 = 0$.